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HEAT TRANSFER IN LAMINAR FLOW OF NON-NEWTONIAN HEAT-GENERATING FLUIDS

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Abstract—The generalized heat balance equation for steady state heat transfer in heat-generating fluids is integrated for laminar flow of a power law non-Newtonian fluid in cylindrical tubes with circular sections, when volumetric heat generation rate depends linearly on local temperature.

The boundary conditions considered are those of arbitrary temperature distribution in the inlet section and constant temperature on the lateral surface of the fluid.

NOMENCLATURE

$a_{i;n}$, constant;
c, specific heat;
c_p, specific heat at constant pressure;
c_v, specific heat at constant volume;
$\frac{D}{Dt}$, substantial derivative;
I_0, modified Bessel function of the first kind of order zero;
m, $= \frac{1}{n} + 1$;
n, constant of power law;
Nu, Nusselt number;
p, pressure;
Q, heat generation rate per unit volume;
Q_0, specific heat generation rate;
r, cylindrical radial co-ordinate;
r_0, tube radius;
t, time;
T, fluid temperature;
T_i, temperature at inlet section centre;
T_0, onset temperature of heat generation;
T_w, wall temperature;
$\mathbf{u}$, velocity vector;
u_0, maximum velocity;
v, volume;
z, cylindrical axial co-ordinate.

Greek symbols
α_n, constant;
β_n, eigenvalue;
η, dimensionless axial co-ordinate;
Θ, dimensionless fluid temperature;
Θ_B, dimensionless bulk temperature;
$\Theta_{\max}$, dimensionless temperature at tube centre;
Θ_1, asymptotic dimensionless temperature at $\eta = \infty$;
Θ_2, $= \Theta - \Theta_1$;
Λ, dimensionless constant of heat generation rate;
λ, heat conductivity;
J_0, Bessel's function of the first kind of order zero;
$\mathbf{j}_q$, heat flux;
ξ, dimensionless radial co-ordinate;
ρ, fluid density;
$\boldsymbol{\tau}$, shear stress tensor;
Ψ_n, eigenfunction.

INTRODUCTION

MANY problems about heat transfer in laminar flow of heat-generating fluids have been studied in recent years [1–21].

The volumetric rate of heat generation was assumed to be constant [1–3, 5, 6, 14, 16, 20] or a function of space variables [8, 10, 11, 12, 15, 18], whereas some authors have considered directly the viscous dissipations and the expansion effect [7, 9, 21].

Topper [4] has obtained solutions for piston flow in pipes with circular cross sections, when the heat generation rate depends linearly on the local temperature.

The authors of the present paper [17] have

x

assumed a volumetric rate of heat generation expressed by the following relation:

$$\left.\begin{array}{ll} Q = Q_0 (T - T_0) & \text{when } T \geqslant T_0 \\ Q = 0 & \text{when } T < T_0 \end{array}\right\} \quad (1)$$

and obtained the steady state temperature profiles for laminar parabolic and piston flow in circular tubes.

Foraboschi and Cocchi [19] examined a particular case of transient conditions, with heat generation as in (1).

The relation (1) may be viewed as an approximation of the rate of some exothermic process increasing with temperature and having T_0 as onset temperature. In this paper the generalized heat balance equation is integrated for steady state heat transfer with arbitrary axially symmetric temperature distribution at the inlet section and constant temperature on the lateral surface of power law non-Newtonian fluid in laminar flow in cylindrical pipes with circular sections, assuming the following heat generation rate:

$$Q = Q_0 (T - T_0). \quad (1')$$

When the inlet wall temperatures are greater than or equal to T_0, the relation (1′) is equivalent to (1).

The radial, axial and mean temperature profiles and the Nusselt numbers, calculated for some values of dimensionless parameters and for constant inlet temperature, are reported.

ANALYTICAL STATEMENT OF THE PROBLEM

The energy equation, neglecting form of energy and energy transport such as electromagnetic, nuclear and radiative, is:

$$\rho\, c_v \frac{DT}{Dt} = - \nabla \,.\, j_q - T \left(\frac{\partial p}{\partial T}\right)_V (\nabla \,.\, \mathbf{u}) - \boldsymbol{\tau} : \nabla\, \mathbf{u} + Q \quad (2)$$

where Q is the heat generation rate per unit volume.

The following assumptions are made:

(a) Fourier's law is valid.
(b) The physical properties of the fluid (ρ, c_v, λ) are constant.
(c) The viscous dissipations are negligible vs. Q.
(d) Axial conduction of heat is negligible in comparison with convective transport.
(e) The velocity profiles are fully developed.
(f) Inlet and boundary conditions have cylindrical symmetry.

Then equation (2), for steady state conditions, reduces to:

$$\rho\, c\, u_z \frac{\partial T}{\partial z} = \frac{\lambda}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + Q \quad (3)$$

where $c_v = c_p = c$.

In fully developed laminar power flow in cylindrical tubes with circular sections the velocity distribution is:

$$u_z = u_0 \left[1 - \left(\frac{r}{r_0}\right)^m\right]. \quad (4)$$

The considered boundary conditions are:

$$\left.\begin{array}{lll} T = T_i\, f(r), & z = 0 & 0 \leqslant r \leqslant r_0 \\ T = T_w, & z > 0 & r = r_0 \end{array}\right\} \quad (5)$$

where $f(0) = 1$, and T_i and T_w are constants.

Substituting (4) into (3) and introducing the following dimensionless variables:

$$\Theta = \frac{T - T_0}{T_i - T_0}, \quad \xi = \frac{r}{r_0}, \quad \eta = \frac{\lambda\, z}{\rho c\, u_0\, r_0^2} \quad (6)$$

we obtain:

$$(1 - \xi^m) \frac{\partial \Theta}{\partial \eta} = \frac{1}{\xi} \frac{\partial}{\partial \xi}\left(\xi \frac{\partial \Theta}{\partial \xi}\right) + \Lambda^2 \Theta \quad (7)$$

in which:

$$\Lambda^2 = \frac{Q_0\, r_0^2}{\lambda} \quad (8)$$

is the dimensionless parameter characterizing the particular heat generation rate.

The dimensionless boundary conditions are:

$$\left.\begin{array}{lll} \Theta = \Theta_i(\xi), & \eta = 0 & 0 \leqslant \xi \leqslant 1 \\ \Theta = \Theta_w, & \eta > 0 & \xi = 1. \end{array}\right\} \quad (9)$$

INTEGRATION OF GENERALIZED HEAT BALANCE EQUATION

Equation (7) with boundary conditions (9) can be solved by stating:

$$\Theta(\eta, \xi) = \Theta_1(\xi) + \Theta_2(\eta, \xi) \quad (10)$$

where Θ_1 is the solution of the following ordinary differential equation:

$$\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\frac{d\Theta_1}{d\xi}\right) + \Lambda^2\,\Theta_1 = 0 \qquad (11)$$

with the boundary conditions:

$$\left.\begin{aligned} &\Theta_1(1) = \Theta_w \\ &\left(\frac{d\Theta_1}{d\xi}\right)_{\xi=0} = 0 \end{aligned}\right\} \qquad (12)$$

·(the second one being a consequence of cylindrical symmetry of temperature distribution), and Θ_2 is the solution of the partial differential equation (7) with the following conditions:

$$\left.\begin{aligned} &\Theta_2(\eta, 1) = 0 \\ &\Theta_2(0, \xi) = \Theta_i(\xi) - \Theta_1(\xi) \\ &\left(\frac{\partial\Theta_2}{\partial\xi}\right)_{\eta,0} = 0. \end{aligned}\right\} \qquad (13)$$

Equation (11) is a Bessel's equation of order zero with parameter Λ, and its solution with boundary conditions (12) is:

$$\Theta_1(\xi) = \Theta_w\frac{J_0(\Lambda\xi)}{J_0(\Lambda)}. \qquad (14)$$

It is interesting to observe that if $\Lambda = 2{\cdot}4048$, then Θ becomes ∞, because $J_0(2{\cdot}4048) = 0$.

Equation (13) may be solved by the method of separation of variables.

The solution is:

$$\Theta_2 = \textstyle\sum_n a_n \exp(-\beta_n^2\,\eta)\,\Psi_n(\xi) \qquad (15)$$

where β_n are the eigenvalues and Ψ_n the eigenfunctions of the following Sturm–Liouville equation:

$$\left.\begin{aligned} &\frac{1}{\xi}\frac{d}{d\xi}\left(\xi\frac{d\Psi_n}{d\xi}\right) + [\Lambda^2 + \beta_n^2(1 - \xi^m)]\,\Psi_n = 0 \\ &\Psi_n(1) = 0, \quad \left(\frac{d\Psi_n}{d\xi}\right)_{\xi=0} = 0. \end{aligned}\right\} \qquad (16)$$

As the eigenfunctions Ψ_n form a system orthogonal with respect to the weight function $\xi(1 - \xi^m)$, then the constants a_n that satisfy the second boundary condition (13) are:

$$a_n = \frac{\int_0^1 \left[\Theta_i(\xi) - \Theta_w\frac{J_0(\Lambda\xi)}{J_0(\Lambda)}\right]\xi(1 - \xi^m)\,\Psi_n\,d\xi}{\int_0^1 \xi(1 - \xi^m)\,\Psi_n^2\,d\xi} \qquad (17)$$

The eigenfunction Ψ_n may be found as power series:

$$\Psi_n = \sum_i^{0\ldots\infty} a_{i;n}\,\xi^i \qquad (18)$$

following the method of Frobenius. A simple solution is obtained for piston flow; indeed in this case $m = \infty$, and the equation (16) reduces to a Bessel's equation of order zero with paramater $\sqrt{(\Lambda^2 + \beta_n^2)}$. Then we obtain:

$$\Psi_n = J_0[\xi\sqrt{(\Lambda^2 + \beta_n^2)}] \qquad (19)$$

and the eigenvalues are determined by the following equation:

$$J_0[\sqrt{(\Lambda^2 + \beta_n^2)}] = 0. \qquad (20)$$

When m is integral it is easily found:

$$a_{i;n} = \frac{1}{i^2}[\beta_n^2\,a_{i-(m+2);n} - (\Lambda^2 + \beta_n^2)\,a_{i-2;n}] \qquad (21)$$

where $a_{0;n}$ may be arbitrarily chosen to be unity and $a_{1;n}$ must be zero to satisfy the condition at $\xi = 0$.

When m is even, every coefficient a_i is equal to 0 whenever i is odd, so (19) and (20) become:

$$\Psi_n = \sum_i^{0\ldots\infty} a_{2i;n}\,\xi^{2i} \qquad (22)$$

$$a_{2i;n} = \frac{1}{(2i)^2}[\beta_n^2\,a_{2i-(m+2);n} - (\Lambda^2 + \beta_n^2)\,a_{2i-2;n}]. \qquad (23)$$

When m is not integral it is necessary to expand ξ^m in power series of ξ, so we may state:

$$\xi^m = \sum_k^{0\ldots\infty}\binom{m}{k}(\xi - 1)^k$$

and then determine the coefficients a_i.

In every case considered the eigenvalues are determined by the equation:

$$\sum_i^{0\ldots\infty} a_i = 0 \qquad (24)$$

following from (18) and the boundary condition $\Psi_n(1) = 0$.

In the case of negative heat generation ($Q_0 < 0$), the equation (7) becomes

$$(1 - \xi^m)\frac{\partial \Theta}{\partial \eta} = \frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi \frac{\partial \Theta}{\partial \xi}\right) - \Lambda^2 \Theta \qquad (7')$$

in which

$$\Lambda^2 = -\frac{Q_0 r_0^2}{\lambda}. \qquad (8')$$

Equation (11) thus becomes a modified Bessel equation of order zero, and with the boundary conditions (12), its solution is

$$\Theta_1(\xi) = \Theta_w \frac{I_0(\Lambda \xi)}{I_0(\Lambda)}. \qquad (14')$$

Because $0 \leqslant \xi \leqslant 1$ and I_0 is a positive and increasing function of ξ, we obtain:

$$\Theta_1(\xi) \leqslant \Theta_w.$$

The general solution is easily obtained by substituting $-\Lambda^2$ to $+\Lambda^2$ wherever $+\Lambda^2$ occurs.

NUMERICAL EXAMPLES

To illustrate the previous solution of generalized Fourier's equation and to show the influence of various parameter, the first eigenvalues and eigenfunctions have been calculated for the following values of Λ and m:

$$\Lambda = \begin{cases} 1 \\ 2 \end{cases} \qquad m = \begin{cases} 2 \\ 4 \\ 6 \\ \infty \end{cases}$$

and for constant inlet temperature T_i.

The first three constants a_n have been calculated with (17) by numerical integration, considering two values of wall temperature, 1 and 3.

The calculated eigenvalues β_n and constants a_n are collected in Table 1, while values of eigenfunctions at various value of the dimensionless radius ξ are given in Table 2.

The axial temperature profiles calculated for the previously indicated values of parameters are shown in Figs. 1, 2, 3 and 4; some of the most interesting radial temperature profiles are shown in Figs. 5 and 6.

The dimensionless mean bulk temperature of fluid in any given section of the pipe may be calculated by using the following integral:

$$\Theta_B = \frac{2(m+2)}{m}\int_0^1 \Theta(1 - \xi^m)\,\xi\,d\xi \qquad (25)$$

and the bulk temperature profiles are shown in Figs. 7, 8, 9 and 10.

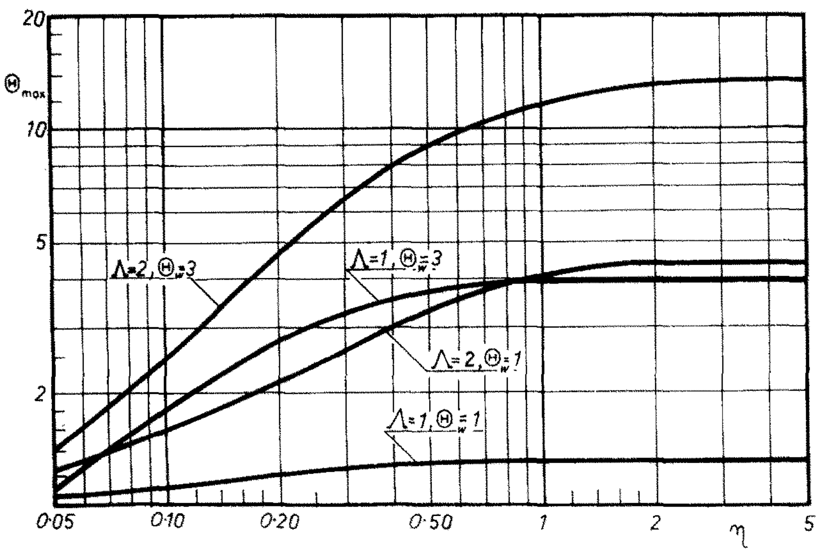

FIG. 1. Axial dimensionless temperature profiles for $m = 2$.

Table 1

m	$\Lambda = 1$									
	β_0	a_0		β_1	a_1		β_2	a_2		β_0
		$\Theta_w = 1$	$\Theta_w = 3$		$\Theta_w = 1$	$\Theta_w = 3$		$\Theta_w = 1$	$\Theta_w = 3$	
2	2·46	−0·3575	−3·9664	6·57	0·0933	1·7297	10·60	−0·0978	−1·2127	1·51
4	2·28	−0·3396	−4·0345	5·94	0·0437	1·8522	9·55	−0·0175	−1·3162	1·40
6	2·23	−0·3397	−4·0809	5·71	0·0424	1·9363	9·18	−0·0115	−1·3862	1·36
∞	2·19	−0·3348	−4·2090	5·43	0·0349	2·2313	8·59	−0·0114	−1·7376	1·34

Table 2

ξ	Ψ_0						Ψ_1			
	$m = 2$		$m = 4$		$m = 6$		$m = 2$		$m = 4$	
	$\Lambda = 1$	$\Lambda = 2$	$\Lambda = 1$	$\Lambda = 2$	$\Lambda = 1$	$\Lambda = 2$	$\Lambda = 1$	$\Lambda = 2$	$\Lambda = 1$	$\Lambda = 2$
0	1	1	1	1	1	1	1	1	1	1
0·1	0·9825	0·9844	0·9845	0·9852	0·9851	0·9854	0·8929	0·8960	0·9113	0·9126
0·2	0·9313	0·9385	0·9388	0·9414	0·9409	0·9423	0·6081	0·6186	0·6688	0·6734
0·3	0·8503	0·8649	0·8649	0·8707	0·8696	0·8726	0·2393	0·2561	0·3361	0·3443
0·4	0·7455	0·7677	0·7667	0·7761	0·7742	0·7793	−0·1039	−0·0861	−0·0013	0·0083
0·5	0·6239	0·6521	0·6491	0·6622	0·6592	0·6664	−0·3382	−0·3257	−0·2631	−0·2551
0·6	0·4932	0·5238	0·5184	0·5341	0·5299	0·5390	−0·4311	−0·4272	−0·3983	−0·3948
0·7	0·3606	0·3891	0·3814	0·3978	0·3925	0·4028	−0·3989	−0·4025	−0·3998	−0·4013
0·8	0·2318	0·2537	0·2451	0·2595	0·2537	0·2636	−0·2867	−0·2932	−0·2998	−0·3043
0·9	0·1111	0·1227	0·1153	0·1249	0·1201	0·1275	−0·1425	−0·1470	−0·1505	−0·1542
1·0	0	0	0	0	0	0	0	0	0	0

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$\Lambda = 2$

α_0		β_1	α_1		β_2	α_2	
$\Theta_w = 1$	$\Theta_w = 3$		$\Theta_w = 1$	$\Theta_w = 3$		$\Theta_w = 1$	$\Theta_w = 3$
−3·6330	−13·7718	6·23	0·2197	2·1491	10·38	−0·0953	−1·3924
−3·6122	−13·8235	5·63	0·1960	2·2645	9·35	−0·0796	−1·4729
−3·6011	−13·8563	5·41	0·1731	2·3057	8·99	−0·0630	−1·5034
−3·6013	−14·0200	5·14	0·1525	2·5793	8·42	−0·0269	−1·7275

		Ψ_2					
$m = 6$		$m = 2$		$m = 4$		$m = 6$	
$\Lambda = 1$	$\Lambda = 2$	$\Lambda = 1$	$\Lambda = 2$	$\Lambda = 1$	$\Lambda = 2$	$\Lambda = 1$	$\Lambda = 2$
1	1	1	1	1	1	1	1
0·9175	0·9184	0·7367	0·7401	0·7823	0·7838	0·7979	0·7993
0·6903	0·6936	0·1554	0·1638	0·2695	0·2737	0·3126	0·3165
0·3738	0·3797	−0·3131	−0·3068	−0·2173	−0·2131	−0·1729	−0·1686
0·0436	0·0509	−0·3932	−0·3961	−0·4044	−0·4044	−0·3991	−0·3983
−0·2253	−0·2189	−0·1461	−0·1566	−0·2491	−0·2542	−0·2930	−0·2973
−0·3791	−0·3759	0·1662	0·1562	0·0618	0·0553	0·0029	0·0039
−0·3998	−0·4010	0·3304	0·3270	0·2854	0·2821	0·2529	0·2487
−0·3096	−0·3142	0·3037	0·3060	0·3058	0·3068	0·3055	0·3066
−0·1581	−0·1632	0·1620	0·1656	0·1728	0·1750	0·1807	0·1851
0	0	0	0	0	0	0	0

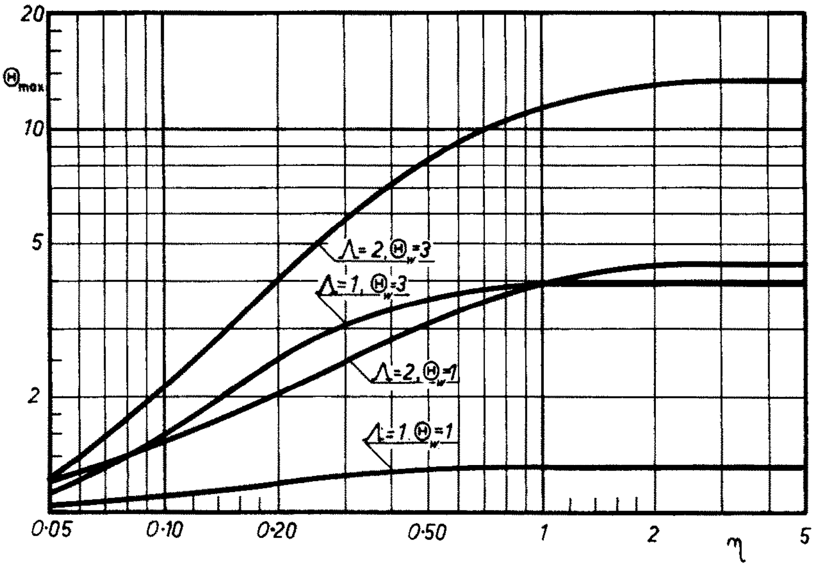

FIG. 2. Axial dimensionless temperature profiles for $m = 4$.

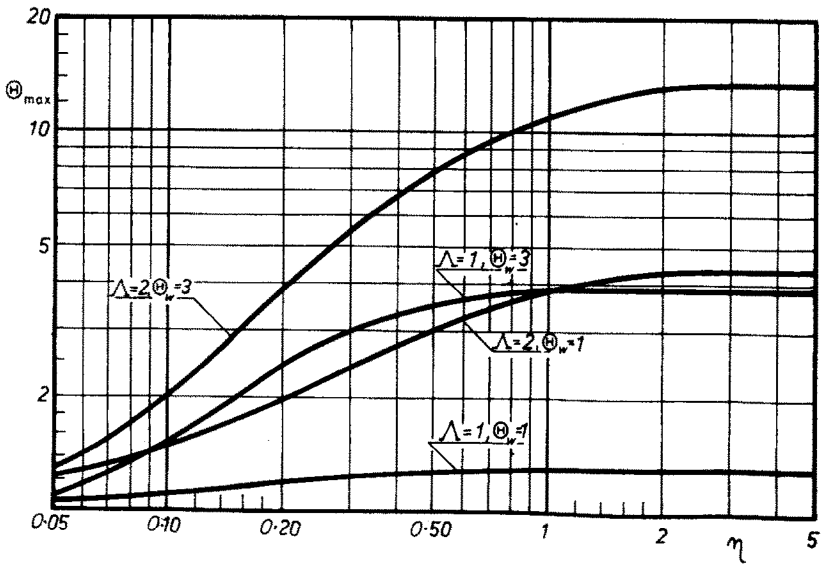

FIG. 3. Axial dimensionless temperature profiles for $m = 6$.

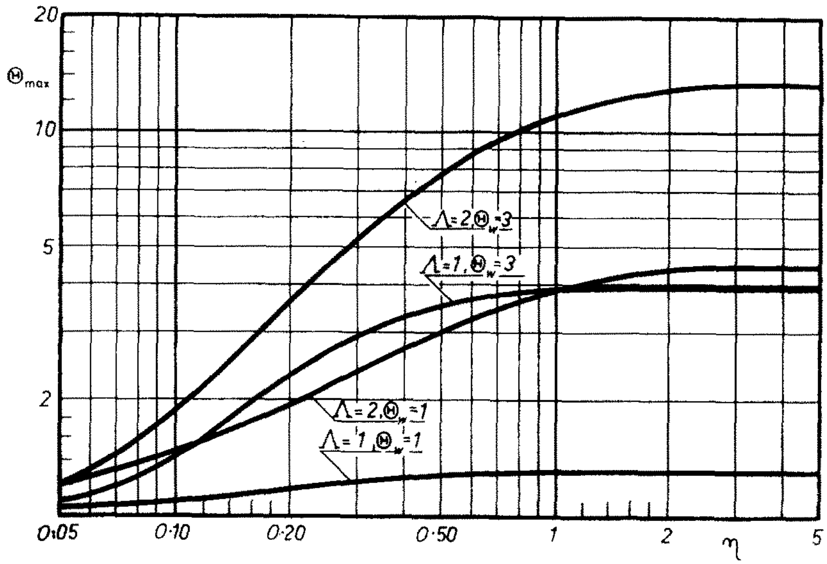

FIG. 4. Axial dimensionless temperature profiles for $m = \infty$.

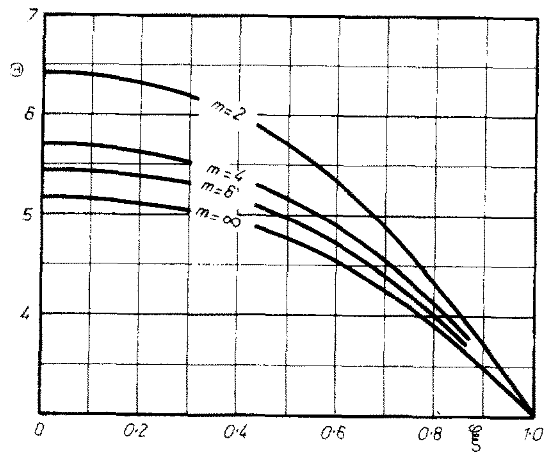

FIG. 5. Radial dimensionless temperature profiles at $\eta = 0{\cdot}3$, for $\Lambda = 2$ and $\Theta_w = 3$.

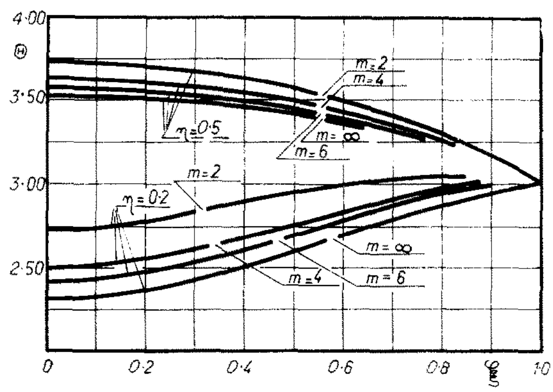

FIG. 6. Radial dimensionless temperature profiles for $\Lambda = 1$ and $\Theta_w = 3$.

The heat transfer at the wall has been calculated as Nusselt numbers:

$$Nu = \frac{2}{\Theta_w - \Theta_B}\left(\frac{\partial\Theta}{\partial\xi}\right)_{\eta;1} \tag{26}$$

and plotted vs. η in Figs. 11, 12, 13 and 14.

The plots of Nusselt numbers show that, when $\Theta_w = 3$, initially Nu decreases and becomes negative, finally attaining a value of $-\infty$; then it changes suddenly becoming $+\infty$, and then decreases quickly reaching values near the asymptote.

This behaviour is easily explained by observing in the plots of bulk and radial temperatures: (a) that for $\Theta_w = 3$, Θ_B becomes greater than Θ_w for those values of η which are higher than those where:

$$\left(\frac{\partial\Theta}{\partial\xi}\right)_{\xi=1}$$

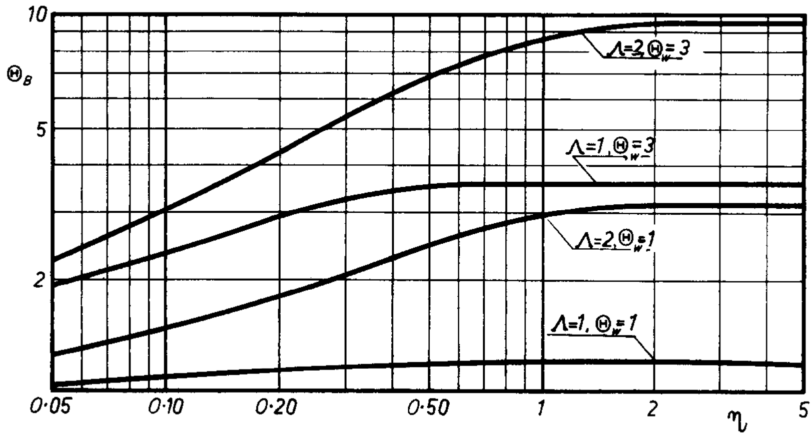

FIG. 7. Dimensionless bulk temperature profiles for $m = 2$.

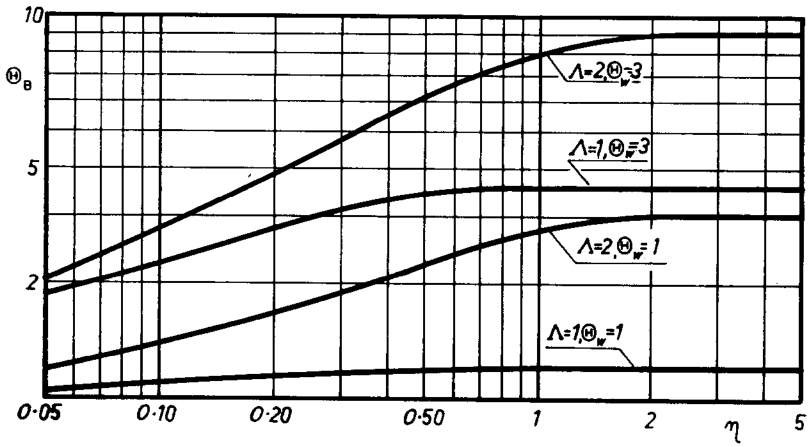

FIG. 8. Dimensionless bulk temperature profiles for $m = 4$.

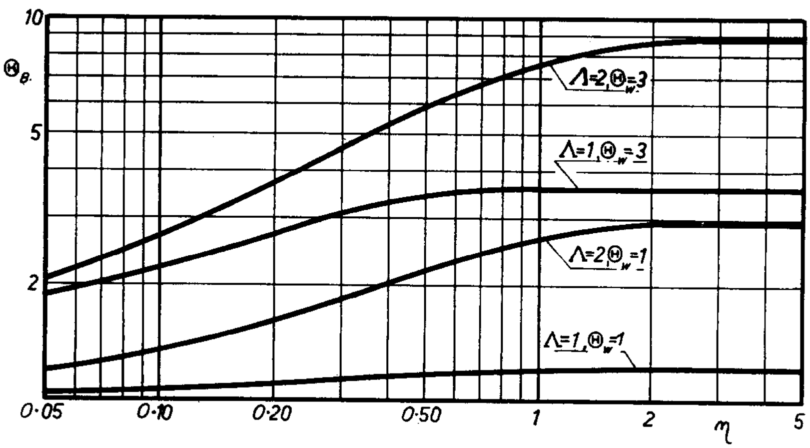

FIG. 9. Dimensionless bulk temperature profiles for $m = 6$.

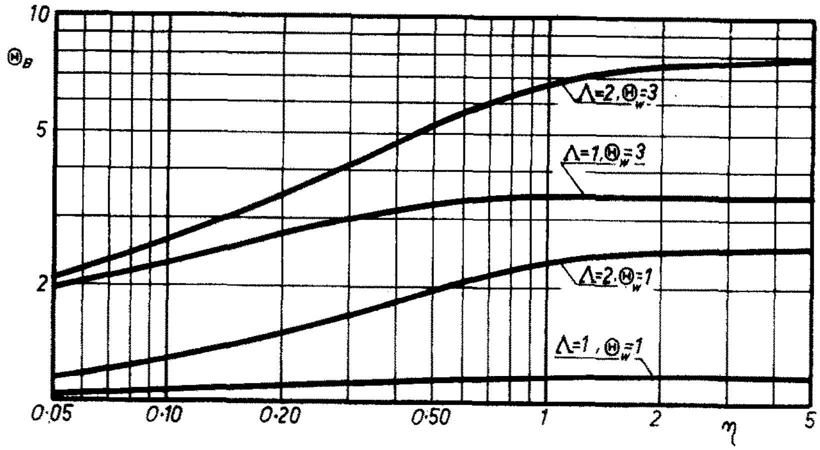

FIG. 10. Dimensionless bulk temperature profiles for $m = \infty$.

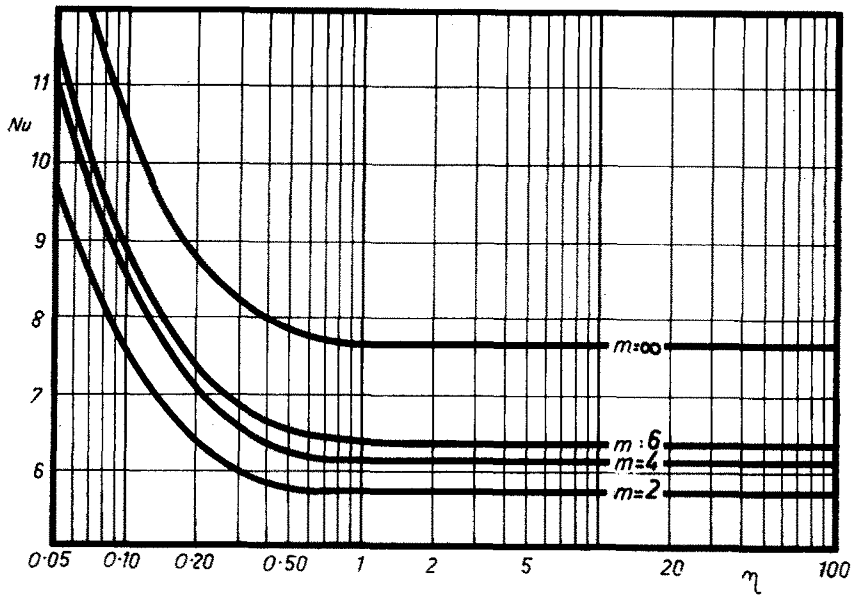

FIG. 11. Nusselt's numbers for $\Lambda = 1$ and $\Theta_w = 1$.

becomes negative, and (b) that when:

$$\Theta_B = \Theta_w, \quad \left(\frac{\partial\Theta}{\partial\xi}\right)_{\xi=1} = 0.$$

CONCLUSION

The results of this investigation show that the generalized heat balance equation for steady state heat transfer in laminar power flow of heat-generating fluids may be integrated when the volumetric heat generation rate depends linearly on temperature, superimposing two solutions.

The first of these is a function of dimensionless radius alone and is the asymptotic temperature reached by the fluid at infinite distance from the inlet section: it is independent of velocity profile and of temperature distribution in the inlet section and depends on Λ and wall temperature. The second one, instead, depends on inlet

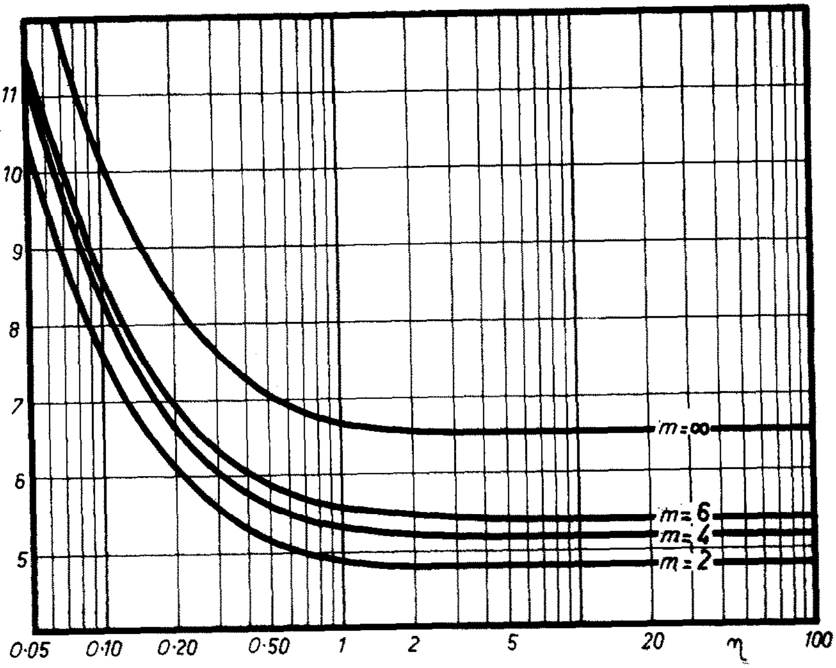

FIG. 12. Nusselt's numbers for $\Lambda = 2$ and $\Theta_w = 1$.

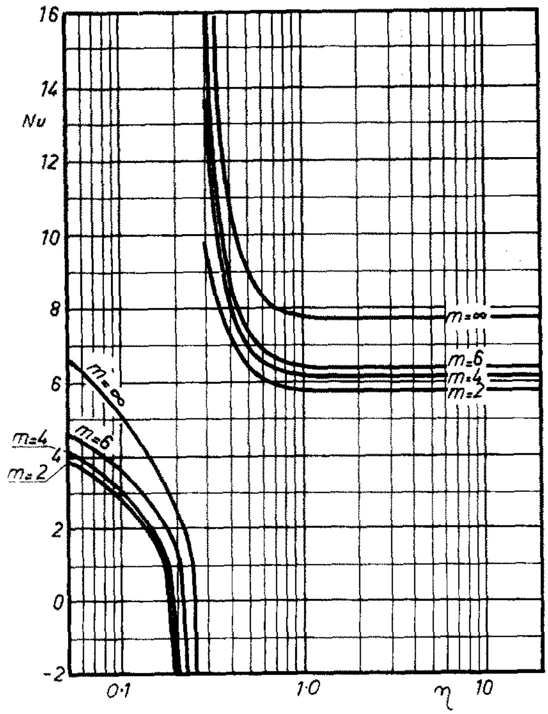

FIG. 13. Nusselt's numbers for $\Lambda = 1$ and $\Theta_w = 3$.

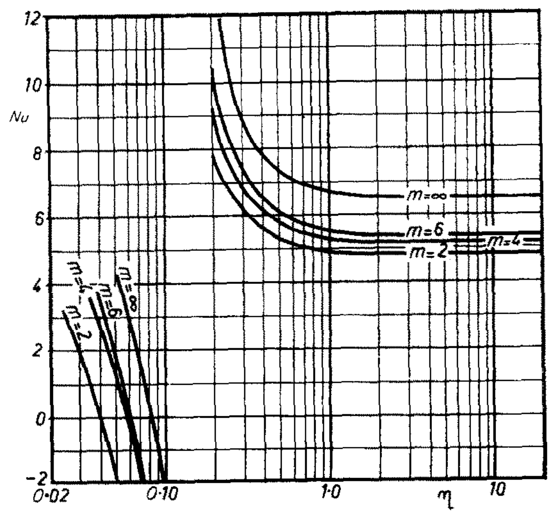

FIG. 14. Nusselt's numbers for $\Lambda = 2$ and $\Theta_w = 3$.

temperature and velocity profiles as well as on Λ and wall temperature.

An interesting result is that for every m it is impossible to have steady state conditions when $\Lambda \geqslant 2{\cdot}4048$.

Instead, when $\Lambda < 2{\cdot}4048$, steady state conditions are reached such that for small values of the axial dimensionless co-ordinate, temperatures and Nusselt numbers differ very little from asymptotic values. In the numerical examples discussed this is verified for $\eta > 1$.

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Résumé—On a intégré l'équation généralisée du bilan de chaleur pour un transfert de chaleur en régime permanent dans des fluides avec production interne de chaleur lorsqu'on a un écoulement laminaire dans des tubes cylindriques à section circulaire d'un fluide non Newtonien obéissant à une loi en puissance, la vitesse de production volumique de chaleur dépendant linéairement de la température locale.

Les conditions aux limites considérées sont celles d'une distribution de température arbitraire dans la section d'entrée et d'une température uniforme sur la surface latérale du fluide.

Zusammenfassung—Die allgemeine Wärmebilanzgleichung für den stationären Wärmeübergang in Medien mit Wärmequellen wird integriert für eine nicht-Newtonsche Flüssigkeit, die einem Potenzgesetz gehorcht und laminar in zylindrischen Rohren mit Kreisquerschnitt strömt. Die volumetrische Wärmeerzeugung hängt linear von der örtlichen Temperatur ab.

Als Grenzbedingungen wurden beliebige Temperaturverteilung im Eintrittsquerschnitt und konstante Temperatur an der begrenzenden Oberfläche der Flüssigkeit zugrundegelegt.

Аннотация—Проинтегрировано обобщенное уравнение теплового баланса для случая стационарного теплообмена в неньютоновских генерирующих тепло жидкостях при их ламинарном течении в цилиндрических круглых трубах. Делались допущения о том, что жидкость подчиняется степенному реологическому закону, а также о линейной зависимости объемной скорости образования тепла от локальной температуры.

Рассматриваемые граничные условия включают в себя произвольное распределение температуры во входном сечении и ее постоянство на боковой поверхности трубы.